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THE UNSTEADY DYNAMIC PROBLEM OF ELECTROELASTICITY FOR AN UNBOUNDED MEDIUM WITH CURVILINEAR TUNNEL CRACKS†

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The behaviour of the stress intensity factors in a piezoelectric medium, weakened by tunnel cracks of fairly arbitrary configuration, is investigated. It is assumed that, in the undeformed state, a crack is associated with a mathematical cut. Normal or shear forces of the impulse type, acting on the edges of the cracks, are considered as the load. The corresponding two-dimensional boundary-value problem of electroelasticity in Fourier transforms is reduced to a system of two singular integral equations for the jumps in displacements on the cuts. The result of calculations are given.

Problems of the impulse excitation of an isotropic medium with a rectilinear finite or semi-infinite crack have been investigated, for example, in [1–3].

1. We consider, in a Cartesian system of coordinates x_1, x_2, x_3, an unbounded piezoelectric medium weakened by tunnel crack-cuts L_j $(j=1, 2, \ldots, n)$ along the x_3 axis. We will agree to assume that the piezoelectric is a transversally isotropic material with the axis of symmetry parallel to the x_3 axis (it is a crystal of the hexagonal system $6mm$, a prepolarized piezoelectric ceramic [4]). We will assume that mechanical forces $X_m(x_1, x_2, t)$ $(m=1, 2)$ which vary arbitrarily with time, and which are independent of the x_3 coordinate, act on the edges of the crack L_j. We will also assume that the curvatures of the contours L_j and $X_m(x_1, x_2, t)$ belong to the class of Hölder functions [5] on $L=\cup L_j$ and, moreover, $\cap L_j=\phi$. Our problem is to determine the parameters of the fracture of the medium with the cracks under unsteady dynamic loading conditions.

In this formulation a state of plane strain occurs in x_1, x_2 in a piezoelectric medium with defects. The complete system of equations has the following form [6]: the equations of state of the piezoelectric medium

$$\begin{gathered}\sigma_{11}=c_{11}\varepsilon_{11}+c_{12}\varepsilon_{22}-h_{31}D_3,\quad \sigma_{22}=c_{12}\varepsilon_{11}+c_{11}\varepsilon_{22}-h_{31}D_3\\ \sigma_{12}=(c_{11}-c_{12})\varepsilon_{12},\quad \sigma_{33}=c_{13}(\varepsilon_{11}+\varepsilon_{22})-h_{33}D_3\\ E_3=-h_{31}(\varepsilon_{11}+\varepsilon_{22})+\beta_{33}D_3\end{gathered}\tag{1.1}$$

the equations of motion

$$\partial_k\sigma_{ik}=\rho_0\partial^2u_i/\partial t^2,\quad \partial_k=\partial/\partial x_k\,(i,k=1,2)\tag{1.2}$$

and Maxwell's equations

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$$\partial_1 E_3 = \mu \partial H_2 / \partial t, \quad \partial_2 E_3 = -\mu \partial H_1 / \partial t \tag{1.3}$$

$$\partial_1 H_2 - \partial_2 H_1 = \partial D_3 / \partial t, \ \partial_1 H_1 + \partial_2 H_2 = 0$$

Here σ_{ik}, ε_{ik}, H_i, E_3 and D_3 are the mechanical stresses and strains, the magnetic and electric field strengths and the electric displacement, respectively, $c_{ij} = c_{ji}^D$ are the moduli of elasticity measured at constant electric induction, h_{31} is the piezoelectric modulus, $\beta = \beta_{33}^{\varepsilon}$ is the dielectric "impermeability", defined for a constant value of the strain, μ is the magnetic permeability of the medium, and ρ_0 is the density of the material. We will assume that there are no external charges and the conductivity of the medium is zero.

It is necessary to add the mechanical boundary conditions on the edges of the cuts to system (1.1)–(1.3). These take the form

$$(\sigma_{11} + \sigma_{22})^{\pm} - e^{2i\psi}(\sigma_{22} - \sigma_{11} + 2i\sigma_{12})^{\pm} = \pm 2e^{i\psi}(X_1^{\pm} - iX_2^{\pm}) \tag{1.4}$$

and also the electromagnetic boundary conditions [7]

$$E_3^+ = E_3^-, \quad H_1^+ = H_1^-, \quad H_2^+ = H_2^- \tag{1.5}$$

The "plus" and "minus" superscripts relate to the left and right edges of L_j when moving from its beginning a_j to its end b_j, and ψ is the angle between the normal to the left edge and the ox_1 axis. Assuming below that the forces on both edges of L_j are self-balancing, we will put $X_m^+ = -X_m^- = X_m (m = 1, 2)$.

2. We will apply a Fourier integral transform with respect to time (with zero initial conditions) to the initial relations (1.1)–(1.3)

$$F(x_1, x_2, \omega) = \frac{1}{\sqrt{2\pi}} \int_0^{\infty} f(x_1, x_2, t) e^{i\omega t} dt \tag{2.1}$$

$$f(x_1, x_2, t) = \sqrt{\frac{2}{\pi}} \operatorname{Re} \int_0^{\infty} F(x_1, x_2, \omega) e^{-i\omega t} d\omega$$

where the function $f(x_1, x_2, t)$ means any of the components of the acoustoelectric field described above, and the parameter ω means the angular frequency.

Taking (2.1) into account in the transformed plane, we can reduce the system of equations (1.1)–(1.3) to two coupled equations in the mechanical displacement vector and the potential $\mathbf{A} = (0, 0, A)$ [8]

$$\nabla^2 \mathbf{U} + \sigma \operatorname{grad} \operatorname{div} \mathbf{U} + \gamma_2^2 \mathbf{U} + \chi \operatorname{grad} A = 0$$

$$\nabla^2 A + k^2 A = \alpha^* \operatorname{div} \mathbf{U}, \quad \mathbf{H}^* = \operatorname{rot} \mathbf{A}, \quad \mathbf{U} = (U_1, U_2)$$

$$\sigma = \frac{\gamma_2^2}{\gamma_1^2} - 1, \quad \gamma_m = \frac{\omega}{c_m}, \quad c_1 = \sqrt{\frac{c_{11} - \kappa_0^2}{\rho_0}}, \quad c_2 = \sqrt{\frac{c_{11} - c_{12}}{2\rho_0}}, \tag{2.2}$$

$$k = \frac{\omega}{c}, \quad c = \sqrt{\frac{\beta_{33}}{\mu}}, \quad \kappa_0^2 = \frac{h_{31}^2}{\beta_{33}}, \quad \chi = \frac{2k^2 h_{31}}{i\omega(c_{11} - c_{12})}, \quad \alpha^* = \frac{i\omega h_{31}}{\beta_{33}} \quad (m = 1, 2)$$

Here $\mathbf{U}$ and $\mathbf{H}^*$ are the Fourier transforms of the displacement vectors $\mathbf{u} = (u_1, u_2)$ and the magnetic intensity $\mathbf{H} = (h_1, H_2)$. The quantity κ_0 represents the piezoelectric effect.

Introducing the representation $\mathbf{U} = \operatorname{grad} \Phi + \operatorname{rot}(\mathbf{k}\Psi)$ ($\mathbf{k}$ is the unit vector along the x_3 axis) into system (2.2), we obtain three differential equations

$$(1+\sigma)\nabla^2\Phi+\gamma_2^2\Phi+\chi A=0$$
$$\nabla^2\Psi+\gamma_2^2\Psi=0,\quad \nabla^2 A+k^2 A=\alpha^*\nabla^2\Phi \tag{2.3}$$

Eliminating A from the first equation of (2.3) and substituting the result into the last equation we obtain

$$(\nabla^2+\beta_1^2)(\nabla^2+\beta_2^2)\Phi=0 \tag{2.4}$$

$$2\beta_{1,2}=\sqrt{(\gamma_1+k)^2+\delta_0^2}\pm\sqrt{(\gamma_1-k)^2+\delta_0^2},\quad \delta_0^2=\kappa_0^2k^2/(c_{11}-\kappa_0^2)$$

It follows from (2.3) and (2.4), in particular, that a monochromatic SV-wave and associated waves of the following types can exist in a piezoelectric material of class $6mm$

$$u_m^{(\nu)}(x_1,\ x_2,\ t)=U_m^{(\nu)}\exp[-i(\omega t+\beta_\nu\mathbf{x}\cdot\mathbf{n})]$$
$$a^{(\nu)}(x_1,\ x_2,\ t)=A^{(\nu)}\exp[-i(\omega t+\beta_\nu\mathbf{x}\cdot\mathbf{n})]$$
$$\mathbf{x}=(x_1,\ x_2),\quad \mathbf{n}=(\cos\beta,\sin\beta)\ (m,\nu=1,2)$$

where $U_m^{(\nu)}$ and $A^{(\nu)}$ are the amplitudes of the displacements and the potential in the wave, characterized by the wave number β_ν ($\nu=1,\ 2$), and β is the angle between the normal to the radiated wavefront and the x_1 axis.

The general solution of Eq. (2.4) has the form

$$\Phi=\Phi_1+\Phi_2 \tag{2.5}$$

where the functions Φ_m ($m=1,\ 2$) are arbitrary solutions of Helmholtz's equation $(\nabla^2+\beta_m^2)\Phi_m=0$.

Integration of the last equation of (2.3) enables us to determine the function A

$$A=-\alpha^*\sum_{m=1}^{2}B_m\Phi_m,\quad B_m=\frac{\beta_m^2}{k^2-\beta_m^2} \tag{2.6}$$

From (2.5), (2.6), (1.1) and (1.3) we can obtain general representations of the acousto-electromagnetic quantities in terms of the functions Ψ and Φ_m ($m=1,\ 2$) in the transformed plane [8]

$$U_1=\partial_1\Phi+\partial_2\Psi,\quad U_2=\partial_2\Phi-\partial_1\Psi$$
$$S_{11}+S_{22}=-\sum_{m=1}^{2}\beta_m^2(c_{11}+c_{12}+2\kappa_0^2B_m)\Phi_m$$
$$S_{22}-S_{11}+2i_{12}S_{12}=-4(c_{11}-c_{12})\frac{\partial^2}{\partial z^2}(\Phi+i\Psi)$$
$$S_{22}-S_{11}-2iS_{12}=-4(c_{11}-c_{12})\frac{\partial^2}{\partial\bar{z}^2}(\Phi-i\Psi) \tag{2.7}$$
$$E_3^*=k^2h_{31}\sum_{m=1}^{2}B_m\Phi_m,\quad D_3^*=\frac{h_{31}}{\beta_{33}}\sum_{m=1}^{2}\beta_m^2B_m\Phi_m$$

$$\mathbf{H}^{*} = \mathrm{rot}\mathbf{A}, \quad z = x_1 + ix_2, \quad \bar{z} = x_1 - ix_2$$

3. Considering the boundary-value problem in the exact formulation, that is, not neglecting, as is usually done, the magnetic field, we will start from (2.7). This approach, taking into account the special integral representations of the potentials, enables us to simplify somewhat the procedure for reducing the initial problem to integral equations. Note that the corresponding boundary-value problem was considered in [6] in the quasi-static approximation. Integral representations of the solutions were used, constructed in [9] by the potential method.

We write the integral representations of the required potentials Φ_ν and Ψ in the form

$$\Phi_\nu(z) = \frac{(-1)^\nu(k^2-\beta_\nu^2)}{\beta_\nu^2(\beta_2^2-\beta_1^2)}\left\{\int_L q_1 H_0^{(1)}(\beta_\nu r)ds + \int_L\left(q_2\frac{\partial H_0^{(1)}(\beta_\nu r)}{\partial\zeta}d\zeta + q_3\frac{\partial H_0^{(1)}(\beta_\nu r)}{\partial\bar{\zeta}}d\bar{\zeta}\right)\right\}$$

$(\nu = 1, 2)$

$$\Psi(z) = \frac{k^2}{i\beta_1^2\beta_2^2}\left\{\int_L q_4 H_0^{(1)}(\gamma_2 r)ds + \int_L\left(q_2\frac{\partial H_0^{(1)}(\gamma_2 r)}{\partial\zeta}d\zeta - q_3\frac{\partial H_0^{(1)}(\gamma_2 r)}{\partial\bar{\zeta}}d\bar{\zeta}\right)\right\} \quad (3.1)$$

$$q_m = q_m(\zeta) = \frac{\gamma_1^2}{8i}(e^{-i\psi}R_1 - (-1)^m e^{i\Psi}R_2) \quad (m = 1, 4)$$

$$q_2 = q_2(\zeta) = i\lambda e^{-i\Psi}R_1'/2, -q_3 = q_3(\zeta) = i\lambda e^{i\Psi}R_2'/2$$

$$R_m = R_m(\zeta) = [U_1] - (-1)^m i[U_2], \quad R_m' = \frac{dR_m}{ds} \quad (m = 1, 2)$$

$$\lambda = \gamma_1^2/\gamma_2^2, \quad r = |\zeta - z|, \quad \zeta = \xi_1 + i\xi_2, \quad \bar{\zeta} = \xi_1 - i\xi_2, \quad \zeta \in L$$

Here ds is an element of the arc of the contour L, and $H_p^{(1)}(x)$ is the Hankel function of the first kind of order p. The square brackets denote jumps in the corresponding quantity on L.

We have the following relations for the functions $q_m(\zeta)$ which follow from (3.1)

$$q_1(\zeta) = \gamma_1^2[U_n]/4i, \quad q_4(\zeta) = \gamma_1^2[U_s]/4$$

$$q_2(\zeta) = i\lambda(V + iV^*)/2, \quad q_3(\zeta) = i\lambda(V - iV^*)/2 \quad (3.2)$$

$$V = [U_n'] - \rho^{-1}[U_s], \quad V^* = [U_s'] + \rho^{-1}[U_n]$$

Here U_n and U_s are the amplitudes of the normal and tangential components, respectively, of the displacement vector on L, and ρ is the radius of curvature of the contour L at the point ζ.

The construction of the representations (3.1) is by no means a trivial operation and requires some explanation. The functions Ψ and Φ_ν must be solutions of the corresponding Helmholtz equations and must satisfy the radiation conditions. In addition, representation (3.1) must ensure that there are jumps in the displacements, that there is continuity of the mechanical-stress vector, and must also ensure that the electromagnetic conditions (1.5) are satisfied on L. By virtue of (2.7), (1.4) and (1.5), these conditions have the form

$$\sum_{\nu=1}^{2}\alpha_\nu[\Phi_\nu] = -4(c_{11} - c_{12})e^{2i\psi}\left[\frac{\partial^2}{\partial z^2}(\Phi + i\Psi)\right]$$

$$e^{2i\psi}\left[\frac{\partial^2}{\partial z^2}(\Phi + i\Psi)\right] = e^{-2i\psi}\left[\frac{\partial^2}{\partial z^2}(\Phi - i\Psi)\right] \quad (3.3)$$

$$\left[\frac{\partial}{\partial z}\sum_{\nu=1}^{2}B_\nu\Phi_\nu\right]=0,\quad \left[\frac{\partial}{\partial \bar z}\sum_{\nu=1}^{2}B_\nu\Phi_\nu\right]=0$$

$$\sum_{\nu=1}^{2}B_\nu[\Phi_\nu]=0,\quad \alpha_\nu=-\beta_\nu^2(c_{11}+c_{12}+2\kappa_0^2B_\nu),\quad z\in L$$

Representations (3.1) were constructed in such a way that conditions (3.3) were automatically satisfied. The remaining two complex functions $[U_n]$ and $[U_s]$ are necessary in order to satisfy the remaining mechanical boundary conditions on the cracks.

4. Boundary conditions (1.4) in Fourier transformants can be written in the form

$$\begin{gathered}(S_{11}+S_{22})^{\pm}-e^{2i\psi}(S_{22}-S_{11}+2iS_{12})^{\pm}=\pm 2e^{i\psi}(X_1^*-iX_2^*)\\ (S_{11}+S_{22})^{\pm}-e^{-2i\psi}(S_{22}-S_{11}-2iS_{12})^{\pm}=\pm 2e^{-i\psi}(X_1^*+iX_2^*)\end{gathered}\tag{4.1}$$

By virtue of the fact that the integral representations (3.1) automatically ensure the continuity of the mechanical-stress vector on L, it is sufficient to satisfy each of conditions (4.1) on only one of the sides of the cut, for example, on the left-hand one. Substituting the limiting values of the functions in (4.1) as $z\to\zeta_0\in L$ we arrive at a system of two complex singular integro-differential equations

$$\sum_{m=1}^{2}\int_L\left\{f_m'(\zeta)G_{m\nu}(\zeta,\zeta_0)+f_m(\zeta)g_{m\nu}(\zeta,\zeta_0)\right\}ds=N_\nu(\zeta_0)\tag{4.2}$$

$$f_1(\zeta)=[U_n],\quad f_2(\zeta)=[U_s],\quad f_m'=\frac{df_m}{ds}\quad(\nu=1,2)$$

$$G_{1\nu}(\zeta,\zeta_0)=i\lambda K_1\sin(\psi-\alpha_0)\pm{1\over 2}(c_{11}-c_{12})e^{\pm 2i\psi_0}(F_1e^{\pm i(\psi-3\alpha_0)}+F_2e^{\mp i(\psi+\alpha_0)})+T_{1\nu}(\zeta,\zeta_0)$$

$$G_{2\nu}(\zeta,\zeta_0)=i\lambda K_1\cos(\psi-\alpha_0)+{1\over 2}i(c_{11}-c_{12})e^{\pm 2i\psi_0}(F_1e^{\pm i(\psi-3\alpha_0)}-F_2e^{\mp i(\psi+\alpha_0)})+T_{2\nu}(\zeta,\zeta_0)$$

$$g_{1\nu}(\zeta,\zeta_0)=-{1\over 2}i\gamma_1^2(K_0+(c_{11}-c_{12})\Phi_{22}e^{\pm 2i(\psi_0-\alpha_0)})+\rho^{-1}G_{2\nu}(\zeta,\zeta_0)$$

$$g_{2\nu}(\zeta,\zeta_0)=\pm{1\over 2}\gamma_2^2(c_{11}-c_{12})H_2(\gamma_2 r)e^{\pm 2i(\psi_0-\alpha_0)}-\rho^{-1}G_{1\nu}(\zeta,\zeta_0),\quad T_{\nu 1}(\zeta,\zeta_0)=T_\nu(\zeta,\zeta_0)$$

$$T_{\nu 2}(\zeta,\zeta_0)=\overline{T_\nu(\zeta,\zeta_0)},$$

$$T_1(\zeta,\zeta_0)=\frac{\lambda}{\pi}(c_{11}+c_{12}-2\kappa_0^2)\left(2\,\mathrm{Im}\frac{e^{i\psi}}{\zeta-\zeta_0}-ie^{2i\psi_0}\frac{e^{-i\psi}+e^{i(\psi-2\alpha_0)}}{\zeta-\zeta_0}\right),$$

$$F_1=\lambda\Phi_{33}+\gamma_2H_3(\gamma_2 r)$$

$$T_2(\zeta,\zeta_0)=\frac{\lambda}{\pi}(c_{11}+c_{12}-2\kappa_0^2)\left(2\,\mathrm{Re}\frac{e^{i\psi}}{\zeta-\zeta_0}-e^{2i\psi_0}\frac{e^{-i\psi}-e^{i(\psi-2\alpha_0)}}{\zeta-\zeta_0}\right),$$

$$F_2=\lambda\Phi_{31}-\gamma_2H_1(\gamma_2 r)$$

$$\Phi_{mn}=(\beta_2^2-\beta_1^2)^{-1}(\beta_2^{m-2}(k^2-\beta_2^2)H_n(\beta_2 r)-\beta_1^{m-2}(k^2-\beta_1^2)H_n(\beta_1 r))$$

$$K_0=c_2^*H_0^{(1)}(\beta_2 r)-c_1^*H_0^{(1)}(\beta_1 r),\quad K_1=c_2^*\beta_2H_1(\beta_2 r)-c_1^*\beta_1H_1(\beta_1 r)$$

$$c_i^*=(\beta_2^2-\beta_1^2)^{-1}(\beta_i^2(c_{11}+c_{12}-2\kappa_0^2)-k^2(c_{11}+c_{12}))$$

$$H_1(x)=\frac{2i}{\pi x}+H_1^{(1)}(x),\quad H_2(x)=\frac{4i}{\pi x^2}+H_2^{(1)}(x)$$

$$H_3(x)=\frac{16i}{\pi x^3}+\frac{2i}{\pi x}+\mp H_3^{(1)}(x),\quad N_\nu(\zeta_0)=4e^{\pm i\psi_0}\mp(X_1^*iX_2^*)$$

$$r=|\zeta-\zeta_0|,\quad \alpha_0=\arg(\zeta-\zeta_0),\quad \psi=\psi(\zeta),\quad \psi_0=\psi(\zeta_0)$$

$$\zeta=\xi_1+i\xi_2,\quad \zeta_0=\xi_{10}+i\xi_{20},\quad \zeta,\zeta_0\in L$$

Here the upper signs correspond to $\nu=1$, and the lower signs correspond to $\nu=2$. The kernels $G_{m\nu}$, $g_{m\nu}$ have a singular character of the Cauchy type due to the functions T_1 and T_2. If the cracks are rectilinear ($\rho=\infty$), the kernel $g_{m\nu}$ is simplified, and by virtue of the assumptions regarding L may possess not more than a logarithmic singularity.

To fix the solution in the class of functions with derivatives that are unbounded on the ends of the contour L_j [5], we must add the following additional conditions to (4.2)

$$\int_{L_j}[U_n']ds=0,\ \int_{L_j}[U_s']ds=0,\ (j=1,2,\ldots,n) \tag{4.3}$$

which express the fact that the jumps in the displacements are equal to zero at the vertices of the cuts.

Hence, the above algorithm for solving the unsteady boundary-value problem for a piezoelectric medium with crack-cuts reduces to the following. From system (4.2), (4.3) we determine the transformants of the jumps in the normal and tangential displacements on the cuts. However, from these functions, using representations (3.1), we establish the potentials Φ_ν ($\nu=1$, 2) and Ψ. Then, from (2.7) in the image plane we determine the acoustoelectromagnetic fields in the medium. To complete the operation one reverts to the originals of the required quantities using (2.1).

5. To determine the dynamic stress intensity factors K_{I} and K_{II} [2], we will obtain the principal asymptotic form of the stresses in the neighbourhood of the tip of the crack. It is convenient to consider the following combinations

$$\begin{aligned} 2(N-iT)&=S_{11}+S_{22}-e^{2i\psi}(S_{22}-S_{11}+2iS_{12})\\ 2(N+iT)&=S_{11}+S_{22}-e^{-2i\psi}(S_{22}-S_{11}-2iS_{12})\end{aligned} \tag{5.1}$$

where N and T are the normal and shear stresses, respectively along the extension to the tip of the defect. Retaining only the singular integrals in (5.1) and using (2.7) and (3.1) we obtain

$$S_{11}^0+S_{22}^0=-\frac{1}{\pi i}(c_{11}+c_{12}-2\kappa_0^2)\left(\int_L\frac{q_2d\zeta}{\zeta-z}+\int_L\frac{q_3d\bar\zeta}{\bar\zeta-\bar z}\right)$$

$$\begin{aligned} S_{22}^0-S_{11}^0+2iS_{12}^0&=\frac{1}{\pi i}(c_{12}-c_{11})\left(\int_L\frac{dR_2}{\zeta-z}-\sigma\int_L\frac{q_2e^{-2i\alpha}d\zeta}{\zeta-z}+(\sigma+2)\int_L\frac{q_3e^{-2i\psi}d\zeta}{\zeta-z}\right)\\ S_{22}^0-S_{11}^0-2iS_{12}^0&=\frac{1}{\pi i}(c_{12}-c_{11})\left(-\int_L\frac{dR_1}{\bar\zeta-\bar z}-\bar\sigma\int_L\frac{q_3e^{-2i\alpha}d\bar\zeta}{\bar\zeta-\bar z}+(\sigma+2)\int_L\frac{q_2e^{-2i\psi}d\bar\zeta}{\bar\zeta-\bar z}\right),\end{aligned} \tag{5.2}$$

$$\alpha=\arg(\zeta-z)$$

The densities R_ν, q_m ($\nu=1$, 2; $m=2$, 3) occurring here are defined in (3.1).

We parametrize the contour L_j: $\zeta=\zeta(\delta)$, $\xi_0=\xi(\delta_0)$, $-1\le\delta$, $\delta_0\le 1$. Correspondingly we put

$$\begin{aligned} &[U_n']=\chi(\delta)\Omega_1(\omega,\delta),\ [U_s']=\chi(\delta)\Omega_2(\omega,\delta)\\ &\chi(\delta)=(s'(\delta)\sqrt{1-\delta^2})^{-1},\quad \Omega_m(\omega,\delta)\in H[-1,1]\quad (m=1,2)\end{aligned} \tag{5.3}$$

By virtue of the formulae describing the behaviour of Cauchy-type integrals in the neighbourhood of the ends of the line of integration [5], and also by virtue of the asymptotic relation [10]

$$\frac{1}{2\pi i}\int_L \frac{\omega(\zeta)e^{-2i\alpha}d\zeta}{\zeta - z} = \pm\frac{e^{\pm i\gamma\pi}\omega^*(c)h(z)}{2i\sin\gamma\pi(z-c)^\gamma} + F_0(z) \tag{5.4}$$

$$\omega(\zeta) = \omega^*(\zeta)(\zeta - c)^{-\gamma}, \quad 0 < \operatorname{Re}\gamma < 1$$

we can conclude that the integrals in (5.2) possess a root singularity at the vertex of the contour c. A complete analysis of (5.2) taking (5.3) and (5.4) into account enables us, after reverting to the originals, to determine the stress intensity factors in the following form (the lower sign refers to the tip $c = b$)

$$K_{\rm I,II}^{\mp} = \mp\frac{\lambda(c_{11}+c_{12}-2\kappa_0^2)}{\sqrt{2s'(\mp 1)}}\operatorname{Re}\int_0^\infty \Omega_{1,2}(\omega,\mp 1)e^{-i\omega t}d\omega \tag{5.5}$$

6. We will consider as our first example a piezoelectric ceramic medium (the material PZT-4 [4]), weakened by a single rectilinear crack of length 21. The associated fields in the medium are excited by a pulsed load of trapezoidal form (Fig. 1). The corresponding spectral functions of this pulse have the form

$$X_v^*(\omega) = \frac{\sigma_v}{\omega^2\sqrt{2\pi}}\left[d_1^{-1}(e^{i\mu_1}-1) - \mu_2(e^{i\omega T} - e^{i\mu_3})\right] \tag{6.1}$$

$$\mu_1 = \omega d_1, \quad \mu_2 = (T - d_1 - d_2)^{-1}, \quad \mu_3 = \omega(d_1 + d_2) \ (v = 1,2)$$

It is convenient to introduce the following dimensionless time parameters (c_2 is the velocity of an SV-wave in the piezoelectric material) $t^* = c_2 t l^{-1}$, $d_i^* = c_2 d_i l^{-1}$, $T^* = c_2 T l^{-1}$.

The functions $\Omega_i(\omega, \mp 1)$ $(i = 1, 2)$ were calculated from system (4.2), (4.3), taking (6.1) into account, using the mechanical-quadrature method [11]. The semi-infinite interval of integration in (5.5) was replaced by a finite interval $[0, \omega^*]$; here the quantity ω^* was found by numerical analysis so as to obtain the minimum error. The number of Chebyshev interpolation nodes on the contour of the cracks was taken to be $N = 15$, 20 and 25, since any further increase in N did not lead to any appreciable increase in the accuracy of the results.

Figure 1 illustrates the change in the quantity $\lambda_1 = K_{\rm I}^+/(\sigma_2\sqrt{(\pi l)})$ as a function of t^* when uniformly distributed normal forces $(X_1 = 0,\ X_2 \neq 0)$ acted on the edges of the crack. The parameters of the pulse were specified to be $T^* = 10$, $d_1 = 1$, $d_2 = 8$, and $\sigma_2^* = c_2\sigma_2 l^{-1} = 1$ N/(s m^2).

As can be seen from Fig. 2, because of the inertial effect, the relative dynamic intensity factor $\lambda_1(t^*)$ may exceed its static value $\lambda_1^0 = 1$ by almost 25%. The fact that in the interval $11c_2^{-1}l < t < 14.6c_2^{-1}l$ the quantity λ_1 takes negative values, expresses the tendency of the edges of the crack to join together a certain time after the load is removed. If uniform tangential forces act on the crack, which vary in accordance with a pulse with parameters $T^* = 10$, $d_1 = 1$, $d_2 = 8$, and $c_1^* = c_2\sigma_1 l^{-1} = 1$ N/(s m^2), then, as numerical investigations show, the dynamic effect for the quantity $\lambda_2(t^*) = K_{\rm II}^+/(\sigma_1\sqrt{(\pi l)})$ (compared with $\lambda_2^0 = 1$) amounts to only 10%.

As a second example we investigated the case of the interaction between two rectilinear cracks L_1 and L_2 with the parametric equations $\xi_1^{(1)} = 3\delta$, $\xi_2^{(1)} = 0$ and $\xi_1^{(2)} = \delta$, $\xi_2^{(2)} = h$. It was assumed that crack L_2 is free from load, while normal pulsed forces with parameters $T^* = c_2 T h^{-1} = 10$, $d_1^* = c_2 d_1 h^{-1} = 1$, $d_2^* = c_2 d_2 h^{-1} = 8$, $\sigma_2^* = C_2\sigma_2 h = 1$ N/(s m^2), act on L_1, where $h/l_2 = 1$. Graphs of $\lambda_1^{(j)} = K_{Ij}^+/(\sigma_2\sqrt{(\pi l_j)})(j = 1, 2$, where $2l_j$ is the length of crack L_j) as a function of $t^* = c_2 t l^{-1}$ are shown in Fig. 2. Curves 1 and 2 relate to cracks L_1 and L_2, respectively.

It follows from the results of the calculations that in a medium with two cracks the inertial effect has greater influence than in a medium with a single crack. As can be seen from Fig. 2, max $|\lambda_1^{(2)}| \approx 2.34$, whereas in the case of a static load $(X_2 = \sigma_2)$ $|\lambda_1^{(2)}| \approx 1.45$.

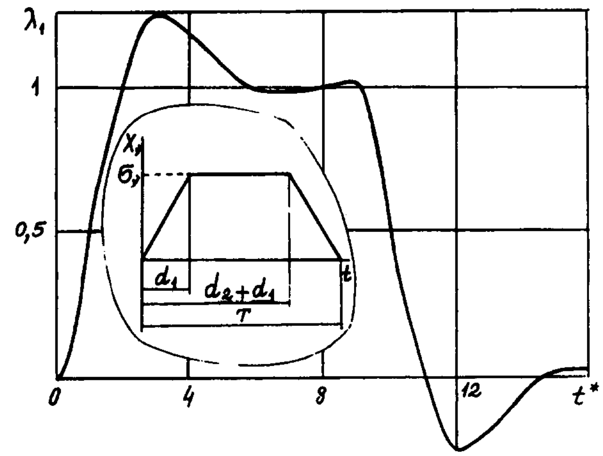

Fig. 1.

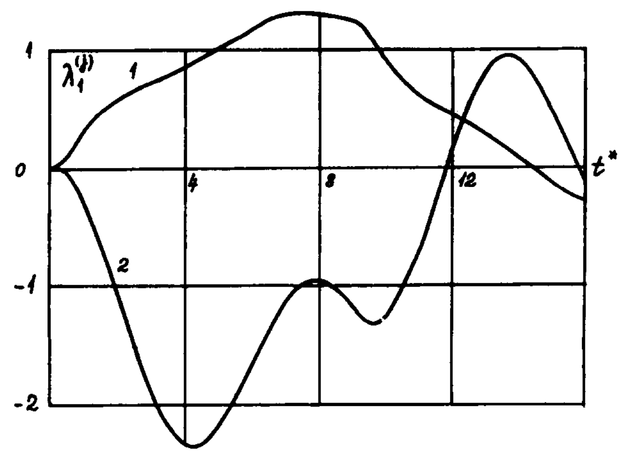

Fig. 2.

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